4.4.2024

N

Galilei-invariant

$\mathbb{R}^{1+3}$

Conservation of momentum

$$\sum_{i=1}^{n} \dot{p}_i = \sum_{i=1}^{n} F_i$$

$$H + L = \dot{x} \cdot p$$

Conservation of information

$$\nabla \cdot \begin{bmatrix} \dot{x} \\ \dot{p} \end{bmatrix} = 0$$

$$\dot{p} = F \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x}$$

$$A[x] = \int L(x, \dot{x})\, dt \to \min !$$

$$\frac{\partial H}{\partial x} = -\dot{p} \qquad \frac{\partial H}{\partial p} = \dot{x} \qquad \{\cdot, H\} = \frac{d}{dt}$$

Conservation of energy

$$\dot{p} = F$$

$$m\ddot{x} = -\partial V$$

$$x(t) = -a t^2$$

Hooke → $x(t) = -\cos \omega t$

magnetic field → $m\ddot{x} = \dot{x} \times B$, $B = \nabla \times A$

$\Longleftarrow$

$$L(x, \dot{x})$$

$$m/2\, \dot{x}^2 - V(x)$$

$$m/2\, \dot{x}^2 - amx$$

$$m/2\, \dot{x}^2 - \tfrac{k}{2} x^2$$

$$\dots$$

$$m/2\, \dot{x} + A \cdot \dot{x}$$

$\Longrightarrow$

$$E = H(x, p)$$

$$p^2/2m + V$$

$$p^2/2m + amx$$

$$p^2/2m + \tfrac{k}{2} x^2$$

$$\dots$$

$$1/2m\, (p - A)^2 = \tfrac{1}{2} m v^2$$

$\Longrightarrow$

QM

$C(\mathbb{R}^{1+3}, \mathbb{C})$

$$\hat{x} = x \qquad \hat{p} = -i\frac{\partial}{\partial x} \qquad \hat{E} = -i\frac{\partial}{\partial t}$$

Heisenberg

$$\Delta\hat{A} \cdot \Delta\hat{B} \geq \tfrac{1}{2}\langle[\hat{A}, \hat{B}]\rangle$$

$$\hat{E} = -i\hat{H}$$ ← Schrödinger

$$\frac{d}{dt} = -\frac{1}{2m}\nabla^2 + V$$

$$d/dt = -\tfrac{1}{2m}\nabla^2 + x^2$$ ← Double slit

$$d/dt = \tfrac{i}{2m}(\nabla - A)^2$$

R

Lorentz-invariant

$$\nabla \cdot p = F \qquad \nabla \cdot \frac{\partial L}{\partial(\nabla\phi)} = \frac{\partial L}{\partial \phi}$$

$$A[\phi] = \int L(\phi, \nabla\phi)\, dx \to \min$$

$$\frac{\partial H}{\partial p} = \nabla\phi \qquad \frac{\partial H}{\partial \phi} = \nabla \cdot p$$

QFT

wave equation → $\partial^2\phi + \partial V = 0$

Maxwell → $\partial F = j$

$\Longleftarrow$

$$-m\sqrt{1 - \dot{x}^2} - V$$

$$1/2\, (\partial\phi)^2 - V(\phi)$$

$$-1/4\, F_{\mu\nu}F^{\mu\nu} + jA = 1/2\, (E^2 - B^2)$$

$\Longrightarrow$

$$E^2 - p^2 = m^2$$

$\Longrightarrow$

$$\partial^2 = m^2$$ ← Gordon-Klein

QED